

Photons and gravitons as Goldstone bosons and the cosmological constant

Per Kraus* and E. T. Tomboulis†

Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547

(Received 24 April 2002; published 19 August 2002)

We reexamine a scenario in which photons and gravitons arise as Goldstone bosons associated with the spontaneous breaking of Lorentz invariance. We study the emergence of Lorentz invariant low energy physics in an effective field theory framework, with non-Lorentz invariant effects arising from radiative corrections and higher order interactions. Spontaneous breaking of the Lorentz group also leads to additional exotic but weakly coupled Goldstone bosons, whose dispersion relations we compute. The usual cosmological constant problem is absent in this context: being a Goldstone boson, the graviton can never develop a potential, and the existence of a flat spacetime solution to the field equations is guaranteed.

DOI: 10.1103/PhysRevD.66.045015

PACS number(s): 11.15.Ex, 11.10.Ef

I. INTRODUCTION

Massless particles can arise by a variety of mechanisms: gauge symmetry, chiral symmetry, supersymmetry, and the spontaneous breaking of global symmetry. The masslessness of the photon and graviton is typically associated with the existence of gauge symmetry; here we explore an alternative option in which it is associated with the spontaneous breaking of Lorentz invariance, with the “gauge” fields arising as Goldstone bosons. This basic idea has a long history, dating back to the 1963 work of Bjorken [1]. For related work, see [2,3,4,5].

At the level of low energy effective field theory, the usual argument in favor of exact gauge invariance is based on Lorentz invariance. Some form of gauge invariance is required in order to obtain an interacting, unitary, Lorentz invariant theory of massless particles with spin 1 or 2. In a manifestly Lorentz invariant formulation a violation of gauge invariance will typically imply a noncancellation of timelike and longitudinal modes, yielding a nonunitary S matrix in the physical sector. In a noncovariant gauge fixed formulation unitarity is manifest, but the action is required to be formally gauge invariant in order to recover Lorentz invariance of the S matrix.

Especially in the case of gravity, there are various motivations for exploring alternatives. First, it is a basic fact that we inhabit a universe that is not Lorentz invariant at large scales due to cosmological expansion. Second, in the case of gravity, gauge invariance is insufficient to guarantee the existence of massless gravitons propagating in Minkowski spacetime, since a potential $\Lambda^4 \sqrt{-g}$ is allowed. Since a non-zero cosmological constant leads to a non-Lorentz invariant vacuum, the original motivation for gauge invariance is not actually realized.

As an alternative we turn to effective actions that yield massless photons and gravitons as Goldstone bosons of spontaneously broken Lorentz invariance. Of course, the effective theory must be consistent with the observed accuracy of Lorentz invariance at distances small compared to the cur-

vature scale of the universe. This is achieved by effective theories consisting of three sorts of terms: gauge invariant kinetic terms, non-gauge invariant potential terms, and small corrections to these. The simplest example is $\mathcal{L} = (F_{\mu\nu})^2 - V(A^\mu A_\mu)$. As we will discuss, it is not too hard to generate such effective actions from some more conventional underlying dynamics. All terms in the action are taken to be Lorentz invariant; however, the potential is assumed to give rise to a constant non-Lorentz invariant vacuum expectation value. The broken Lorentz generators imply the existence of massless Goldstone bosons. In the absence of the potential, the vacuum expectation value would have no physical effect, being pure gauge; in particular, exact Lorentz invariance would be maintained. With the potential included gauge invariance is broken and with it exact Lorentz invariance. However, by definition the Goldstone bosons do not appear in the potential; the expansion of the potential is in terms of massive fields. Since Lorentz invariance is broken only by the potential, the breaking will be suppressed by the inverse mass of the heavy fields. So at low energies we will have an approximately Lorentz invariant theory of Goldstone bosons. The expansion of the potential to quadratic order in fluctuations simply acts a gauge fixing term, so the low energy theory is approximately that of photons or gravitons in a noncovariant gauge.

Actually, there are additional interesting complications in the quantum theory, where loops induce small non-Lorentz invariant kinetic terms. This leads to the appearance of additional non-Lorentz invariant but “weakly coupled” Goldstone bosons whose effects we discuss. Directly related to this is that it is crucial to study whether the form of our effective action is stable under radiative corrections, as these will generically induce all possible operators consistent with the symmetries. We study this question carefully, and find that with reasonable starting assumptions the resulting low energy physics appears approximately Lorentz invariant.

Motivated by the cosmological constant problem, a multitude of authors have experimented with modifying gravity in various ways (see [6] for a review of some attempts). From the point of view of low energy effective field theory the problem is that general covariance allows one—and only one—potential term, $\Lambda^4 \sqrt{-g}$, and so unless this term vanishes there exists no solution to the equations of motion with constant fields. But in a scenario in which the graviton is a

*Email address: pkraus@physics.ucla.edu

†Email address: tombouli@physics.ucla.edu

Goldstone boson this problem does not arise, since a Goldstone boson can never acquire a potential. So even if some scalar field undergoes a phase transition, contributing a term $V(\phi_0)\sqrt{-g}$ to the effective action, one knows that the vacuum expectation value for massive fields can be shifted such that the Goldstone boson gravitons remain massless. Therefore, there will *always* exist an exact vacuum solution with constant fields, on which propagate massless gravitons with approximately Lorentz invariant physics. This then guarantees the existence of the sort of solution one wants without fine-tuning. However, one should note that there may also exist other solutions with spacetime varying fields. A complete solution to the problem should address why a flat (or nearly flat) spacetime solution is preferred; we discuss this in Sec. III.

The first part of this paper is devoted to the detailed study of the photon as a Goldstone boson, but this is essentially a warmup for the more interesting case of gravity, which provides our main motivation. There does not seem to be any obvious advantage in producing the photon as a Goldstone boson, and we have made no attempt at a realistic model by including the other standard model fields. We should also stress that we will work in an effective field theory framework in which the graviton is to be thought of as a composite of more fundamental degrees of freedom. It may be worth mentioning why our scenario is not in conflict with the theorem of Weinberg and Witten [7], which rules out “composite gravitons” in a broad class of models. Specifically, the theorem states that a Lorentz invariant theory with a Lorentz invariant vacuum and a Lorentz covariant energy-momentum tensor cannot have a massless spin 2 particle in its spectrum. There is no conflict here since our vacuum will not be Lorentz invariant.

This paper is organized as follows. In Sec. II we study in detail the example of the photon as a Goldstone boson. The photon example illustrates most of the important general features and is much simpler computationally than the gravity case. We also discuss the relation to previous work on this subject. In Sec. III we discuss the graviton and the cosmological constant problem in this context. Section IV has some final comments, and the Appendix contains technical results for the gravity case.

II. PHOTON AS GOLDSTONE BOSON

We begin by writing down a certain effective action for a vector field A_μ coupled to matter. The motivation for this form will be discussed subsequently. The action is to be thought of as an effective action defined at a UV cutoff scale Λ . We thus consider the Lagrangian

$$\begin{aligned} \mathcal{L} = N \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(A^\mu A_\mu) \right. \\ \left. + \text{higher derivatives} \right\} + \mathcal{L}_{\text{matter}}(\phi, A_\mu) \\ + \mathcal{O}(N^0). \end{aligned} \quad (2.1)$$

In the above, N is some large number, and we have written out the leading N terms in the action. The action is Lorentz invariant—all indices being raised and lowered with the Minkowski metric—but not gauge invariant. In particular, the potential $V(A^\mu A_\mu)$ is not gauge invariant; a crucial point is that this is the *only* non-gauge invariant term at leading order in N . In particular, the higher derivatives terms are terms like $(F^{\mu\nu} F_{\mu\nu})^2$, $\partial^\alpha F^{\mu\nu} \partial_\alpha F_{\mu\nu}$, etc. Similarly, the generic matter fields ϕ are gauge invariantly coupled to A_μ in $\mathcal{L}_{\text{matter}}(\phi, A_\mu)$. Apart from these stipulations the action is generic in the sense that all dimensionless couplings (apart from N) are of order unity; that is, all dimensionful quantities are of the order of the cutoff Λ to the appropriate power.

Since in our effective action we have explicitly excluded various terms which are consistent with the symmetries of the theory, two questions immediately come to the fore: how might an action of this form be generated from some underlying dynamics, and is the assumed structure of the action stable under radiative corrections? We now address these two points in turn.

A. Effective action from fermions

To show why an action of the form (2.1) is fairly natural, we show how it can be generated by integrating out some large number of fermion species. This will be a generalization of the original mechanism proposed by Bjorken [1], which considered four fermion interactions. So consider N species of Dirac fermions ψ_i . We imagine these fermions being coupled to gauge fields which acquire masses at scale Λ . Integrating out the massive gauge bosons will yield an infinite set of fermion interactions, and we will focus on the following subset:

$$\mathcal{L}_\psi = \bar{\psi}_i (i \not{\partial} - m) \psi_i + N \sum_{n=1}^{\infty} \lambda_{2n} \frac{(\bar{\psi}_i \gamma^\mu \psi_i)^{2n}}{N^{2n}}. \quad (2.2)$$

Here, summation over flavor indices i and spacetime indices μ is implied. We wrote the action to have a $U(N)$ flavor symmetry. The couplings λ_{2n} are of order unity times the appropriate power of Λ ,

is defined by a path integral (with a cutoff) over the fields ψ_i and A^μ . The idea is to imagine first doing the path integral over ψ_i to yield an effective action for A^μ . Since ψ_i is minimally coupled to A^μ , provided we choose a gauge invariant cutoff the terms in the effective action generated in this way will be gauge invariant. Furthermore, since there are N species of fermions the effective action will have an overall factor of N . Therefore, the form of the effective action is that of the first set of terms in Eq. (2.1).

Now suppose we had included the other fermion terms in Eq. (2.2) that would certainly arise upon integrating out massive gauge fields. This will introduce other bilinears in the theory, e.g., $\bar{\psi}_i \gamma^{\mu\nu} \psi_i$, $\bar{\psi}_i \gamma^\mu \partial^\nu \psi_i$, etc. The above procedure should then be generalized by introducing a new auxiliary field for each bilinear. Integrating out the fermions then yields an effective action for a set of interacting auxiliary fields. The analysis rapidly becomes complicated; however, one expects on general grounds that the auxiliary fields will acquire mass terms of order of the cutoff and so can be neglected at lower energies. By contrast, in our scenario certain components of A^μ will remain massless since they will correspond to Goldstone bosons.

To reiterate somewhat, in our approach where we consider A^μ as the only auxiliary field it is consistent to omit terms like $(\bar{\psi}_i \gamma^\mu \psi_i)^2 \bar{\psi}_j \partial^\nu \psi_j$ which might seem to lead to non-gauge invariant terms like $f(A^\mu A_\mu) F^{\alpha\beta} F_{\alpha\beta}$ in the effective action for A^μ . If such fermionic terms are to be included one should introduce a new scalar auxiliary field for the bilinear $\bar{\psi}_i \partial^\nu \psi_j$, and we are not doing this for the reasons stated above.

We have thus demonstrated one possible way of generating an effective action with the structure (2.1), though there are presumably other ways as well. For the most part we consider Eq. (2.1) in its own right, without reference to its origin.

B. Coupling to matter

By coupling a matter field ϕ to the fermions via conserved currents, $J^\mu(\phi) \bar{\psi}_i \gamma_\mu \psi_i$, we will generate the matter couplings given in Eq. (2.1). An easy way to accomplish this is to modify Eq. (2.2) by taking the mass for some of the fermions to be much less than the cutoff Λ . In this case we would keep the light fermions in the low energy effective action rather than integrating them out. From Eq. (2.4) we see that these fermions will be minimally coupled to A^μ . Whether we use this or some other mechanism to generate the matter couplings, it will be important that the matter action is gauge invariant, at least up to the level of two derivative terms.

C. Stability under radiative corrections

Treating Eq. (2.1) as an effective field theory at scale Λ , to extract out low energy physics we still need to integrate out the fluctuations of A^μ . Since the potential term violates gauge invariance, once we start computing loop diagrams all possible Lorentz invariant, but not necessarily gauge invariant, terms will generically be generated. Some of these terms would lead, after spontaneous symmetry breaking, to large

violations of Lorentz invariance at low energies and so need to be suppressed. The need to suppress such terms is our motivation for introducing the large number N . Given the form (2.1), the loop expansion is an expansion in $1/N$, so the dangerous terms will only arise at order N^0 .

So at order N^0 we need to consider all possible Lorentz invariant terms generated by computing loop diagrams for A^μ . At energies low compared to the cutoff we can restrict attention to terms with at most two derivatives. We further assume symmetry under charge conjugation C , acting as sign reversal on A^μ . This forbids single derivative terms. Terms with no derivatives just give a small correction to the potential in Eq. (2.1), which we are taking to be arbitrary, so we need consider only two derivative terms. Up to integration by parts, there are seven independent terms:

$$\begin{aligned} (1) \quad & f_1(A^2) \partial_\mu A_\nu \partial^\mu A^\nu, \\ (2) \quad & f_2(A^2) \partial_\mu A_\nu \partial^\nu A^\mu, \\ (3) \quad & f_3(A^2) A^\mu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\nu, \\ (4) \quad & f_4(A^2) A^\nu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\mu, \\ (5) \quad & f_5(A^2) A^\nu A^\alpha \partial_\mu A_\nu \partial^\mu A_\alpha, \\ (6) \quad & f_6(A^2) A^\mu A^\nu A^\alpha \partial_\mu \partial_\nu A_\alpha, \\ (7) \quad & f_7(A^2) A^\mu A^\nu A^\alpha A^\beta \partial_\mu A_\nu \partial_\alpha A_\beta. \end{aligned} \quad (2.5)$$

Here $A^2 \equiv A^\mu A_\mu$. As always, we assume that all dimensionful

$$V = V(-c^2\Lambda^2) + \frac{1}{2\alpha}(n \cdot a)^2 + \dots, \quad (2.9)$$

where

$$\alpha = \frac{1}{4V''(-c^2\Lambda^2)} \sim \frac{1}{\Lambda^2}. \quad (2.10)$$

Shifting the vacuum has a trivial effect on the A^μ kinetic terms since, being gauge invariant, these depend only on derivatives of A^μ . Hence in the kinetic terms we can just replace $A^\mu \rightarrow a^\mu$. We can similarly make this replacement in the matter Lagrangian after performing a compensating gauge rotation of the matter fields:

$$\mathcal{L}_{\text{matter}}(\phi, \langle A_\mu \rangle + a_\mu) = \mathcal{L}_{\text{matter}}(\phi', a_\mu), \quad (2.11)$$

where ϕ' is a gauge transformation of ϕ . We will henceforth drop the prime on ϕ . Therefore, the action takes the form

$$\begin{aligned} \mathcal{L} = N \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\alpha} (n \cdot a)^2 \right. \\ \left. + \text{higher derivatives} + \mathcal{O}(a^3) \right\} \\ + \mathcal{L}_{\text{matter}}(\phi, a_\mu) + \mathcal{O}(N^0). \end{aligned} \quad (2.12)$$

The $(n \cdot a)^2$ term plays the role of an axial gauge fixing term. It gives a mass of order Λ to a spacelike component of a_μ . So to the above order our action takes the form of an axial gauge fixed photon coupled gauge invariantly to matter. Neglecting the higher derivative terms, the photon propagator is given by

$$\begin{aligned} -\frac{i}{Np^2} \left(\eta_{\mu\nu} - \frac{1}{n \cdot p} (n_\mu p_\nu + n_\nu p_\mu) \right. \\ \left. - \frac{p_\mu p_\nu}{(n \cdot p)^2} (\alpha p^2 - n^2) \right). \end{aligned} \quad (2.13)$$

As usual, only the first term contributes when the propagator is sandwiched between conserved current, and the result is Lorentz invariant. Corrections to this Lorentz invariant result are suppressed by p/Λ and/or $1/N$, as will be discussed below.

E. Goldstone bosons

After spontaneous symmetry breaking we have “massless particles”¹ by virtue of Goldstone’s theorem. In particular, a spacelike vector breaks the Lorentz group according to

$$\text{SO}(3,1) \rightarrow \text{SO}(2,1). \quad (2.14)$$

¹We employ quotation marks since we can no longer use the standard definition of particles as being irreducible representations of the (now spontaneously broken) Poincaré group. But it should be clear what we mean when we use the particle terminology.

There are thus three broken generators corresponding to two rotations and a boost. The corresponding three Goldstone bosons are the three components of a^μ orthogonal to n^μ . We choose a basis of two transverse components and a timelike component:

$$\text{transverse: } \epsilon_\mu^{(1,2)}, \quad k \cdot \epsilon^{(1,2)} = n \cdot \epsilon^{(1,2)} = 0,$$

$$\text{timelike: } \epsilon_\mu^{(0)} = k_\mu + (n \cdot k) n_\mu,$$

$$\text{obeys } n \cdot \epsilon^{(0)} = 0. \quad (2.15)$$

At low energies only the Goldstone bosons are relevant. At leading order in N the gauge invariant form of the kinetic terms implies that only the transverse Goldstone bosons propagate, giving us the conventional Lorentz invariant electrodynamics. However, at order N^0 the timelike component will also propagate, and this will lead to interesting effects.

We now concentrate on the low energy physics of the Goldstone bosons coupled to matter. The Goldstone bosons can be thought of as coordinates on the coset $\text{SO}(3,1)/\text{SO}(2,1)$. Since the Goldstone bosons label flat directions of the potential, the cubic and higher order terms from the expansion of the potential all involve the massive component $n \cdot a$, and so are irrelevant at low energies. Now consider the order N^0 terms (2.

rents we will get the standard QED results at leading order, with non-Lorentz invariant corrections occurring at order $1/N$.

F. Low energy spectrum

We now determine the dispersion relations for the three Goldstone bosons by solving the linearized equations of motion. The latter are

$$\left(1 + \frac{c_1}{N}\right) \partial^\mu \partial_\mu a^\nu - \partial^\nu \partial_\mu a^\mu + \frac{c_2}{N} n^\alpha n^\beta \partial_\alpha \partial_\beta a^\nu - \frac{1}{\alpha} n^\mu a_\mu n^\nu = 0. \quad (2.18)$$

First consider the transverse modes. Using the ansatz [see Eq. (2.15)]

$$a_\nu = \epsilon_\nu^{(1,2)} e^{-ik \cdot x} \quad (2.19)$$

we find the dispersion relation

$$\text{transverse: } \left(1 + \frac{c_1}{N}\right) k^2 + \frac{c_2}{N} (n \cdot k)^2 = 0. \quad (2.20)$$

This corresponds to an anisotropic speed of light. The speed of light parallel to n^μ differs from that orthogonal to n^μ by an amount of order $1/\sqrt{N}$.

Now consider the timelike mode. We use the ansatz

$$a_\nu = (k_\nu + \gamma n_\nu) e^{-ik \cdot x}. \quad (2.21)$$

We find

$$\gamma = n \cdot k + \mathcal{O}(\alpha k^2), \quad (2.22)$$

and the dispersion relation

$$\text{timelike: } k^2 - \frac{N}{c_1} (n \cdot k)^2 = 0. \quad (2.23)$$

In the dispersion relation we have dropped terms down by $1/N$ or αk^2 . The dispersion relation (2.23) is non-Lorentz invariant at leading order. The timelike modes propagate at the ordinary speed of light in directions orthogonal to n^μ , but at a speed of order $\sqrt{N}$ in the direction parallel to n^μ . We are assuming that $c_1 > 0$.

The physics of the transverse modes is thus standard up to small corrections, while that of the timelike mode is quite exotic. The reason for this is that the timelike mode does not propagate with respect to the leading N gauge invariant kinetic terms of Eq. (2.1). It only acquires a kinetic term at order N^0 , and these terms are non-Lorentz invariant after spontaneous symmetry breaking.

While exotic, the timelike mode leads to acceptably small effects for sufficiently large N . Its contribution to the interaction between conserved currents is suppressed by $1/N$, since as we have discussed we can use the standard axial

gauge propagator at leading order and regard corrections as coming from interaction vertices with coefficients of order $1/N$.

The timelike mode is also suppressed by phase space considerations. For fixed available energy k^0 the dispersion relation forces

$$|k \cdot n| < \sqrt{c_1/N} k^0. \quad (2.24)$$

Consider putting the system in a box of size L . Then for $N > (k^0 L)^2$ only the zero momentum mode parallel to n^μ survives, yielding a phase space suppression proportional to $1/L$.

While the timelike mode gives small corrections to the interaction between conserved currents it could have more dramatic consequences given its unusual dispersion relation. We should emphasize that the result (2.23) does not necessarily imply faster than light² signal propagation, since Eq. (2.23) is only valid for long wavelengths. Also, even if Eq. (2.23) could be extrapolated to short wavelengths so that signals could propagate faster than light, there would be no conflict with causality since Lorentz invariance has been spontaneously broken by a preferred frame. It would be interesting to study the physics of the timelike mode in more detail.

G. Summary and relation to previous work

gent integrals. The claim was then made that the physics after spontaneous symmetry breaking was the usual *exactly* Lorentz invariant quantum electrodynamics. This conclusion was again dependent on manipulating divergent quantities. As far as we can tell, the origin of this claim is that if one takes the pure four Fermi interaction and assumes (wrongly) that this leads to spontaneous symmetry breaking, then expanding $V \sim A^\mu A_\mu$ around the new vacuum would yield an axial gauge fixing term and nothing more. This of course gives the usual QED in the axial gauge. But from our point of view it is clear what would actually happen in this theory. The four Fermi theory leads either to an instability or to a stable vacuum with a massive vector field A_μ . In neither case does one find QED. On the other hand, the problem disappears once one includes the higher order fermion terms as we have done; this is also the natural starting point from the view of effective field theory.

Since earlier work took the point of view that the spontaneous breaking of Lorentz invariance was somehow fictitious, the existence of the extra Goldstone boson was not noted.

Before turning to gravity we should also note that some of the above criticisms were commented on recently by Bjorken [8]. In particular it was noted that the four Fermi theory by itself is inadequate, and that some real violations of Lorentz invariance should be expected once quantum effects are taken into account. These points were also examined by Banks and Zaks in [9] in the context of non-Abelian gauge fields; they also concluded that there is a real violation of Lorentz invariance. We hope to have resolved these issues here.

III. GRAVITON AS GOLDSTONE BOSON

We now turn to our main interest: producing a graviton as a Goldstone boson. Fortunately, the analysis closely parallels the photon case, and so we can draw on our experience from that example to navigate in the more complicated gravitational setting. The main difference is in the different pattern of spontaneous Lorentz breaking as well as in the connection to the cosmological constant problem. The cosmological constant provides a motivation for modifying the low energy effective theory of general relativity, and indeed we will see that the problem is avoided in the sense that the Goldstone boson graviton remains massless even in the presence of vacuum energy.

To adapt the previous approach to the case of gravity we consider an effective action in direct analogy with Eq. (2.1),

$$\begin{aligned} \mathcal{L} = & N \{ \Lambda^2 \sqrt{-g} R(g) - \Lambda^4 V(h) \\ & + \text{higher derivatives} \} + \mathcal{L}_{\text{matter}}(\phi, g) \\ & + \mathcal{O}(N^0). \end{aligned} \quad (3.1)$$

Here h is defined via expansion of the metric around flat spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (3.2)$$

Flat spacetime thus plays a preferred role in this context; indeed we think of the underlying dynamics as that of a nongravitational theory in Minkowski spacetime. The Appendix discusses one possibility for such an underlying theory. The Einstein-Hilbert term in Eq. (3.1) is standard while the potential is some generic Lorentz invariant function of $h_{\mu\nu}$ with indices contracted with $\eta_{\mu\nu}$. Odd powers of h are allowed; for instance, the term h^μ_μ can appear in the expansion of $V(h)$. As in the photon case the higher derivative terms are generally covariant, as is the matter action. General covariance is violated only by the potential.

Note that the observed Newton's constant will be

$$G_N \sim \frac{1}{N\Lambda^2}. \quad (3.3)$$

Therefore, the cutoff Λ is smaller by a factor of $1/\sqrt{N}$ compared to the usual Planck scale. Indeed, this lowered value of the cutoff is responsible for suppressing loop corrections. N is a large number but presumably need not be more than 10^4 or so. Thus the cutoff can still be well above observable energy scales.

We consider potentials leading to a vacuum expectation value for $h_{\mu\nu}$. By performing a Lorentz transformation we can bring the expectation value to the form

$$\langle h_{\mu\nu} \rangle = \begin{pmatrix} \bar{h}_{00} & & & \\ & \bar{h}_{11} & & \\ & & \bar{h}_{22} & \\ & & & \bar{h}_{33} \end{pmatrix}. \quad (3.4)$$

For $\bar{h}_{\mu\mu}$ all nonvanishing and distinct, the Lorentz group will be completely broken:

$$\text{SO}(3,1) \rightarrow \text{nothing}. \quad (3.5)$$

Being dimensionless, we expect $\bar{h}_{\mu\mu}$ of order unity. Therefore there will be six Gold

$$V(h) = \text{const} + \Lambda^4 \sum_{\alpha=0}^3 (f_{\alpha} n_{(\alpha)}^{\mu} n_{(\alpha)}^{\nu} \tilde{h}_{\mu\nu})^2 + \mathcal{O}(\tilde{h}^3). \quad (3.7)$$

Here

$$n_{(\alpha)}^{\mu} = \delta_{\alpha}^{\mu}, \quad (3.8)$$

and f_{α} are numbers of order unity.

We can simplify the action by performing a general coordinate transformation to put the background metric back in standard form. In particular, introduce new coordinates x'^{μ} such that

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x'^{\nu}}{\partial x^{\beta}} \eta_{\mu\nu} = \eta_{\alpha\beta} + \langle h_{\alpha\beta} \rangle. \quad (3.9)$$

The metric appearing in the action will then be $\eta_{\mu\nu} + \tilde{h}'_{\mu\nu}$ where

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x'^{\nu}}{\partial x^{\beta}} \tilde{h}'_{\mu\nu}(x') = \tilde{h}_{\alpha\beta}(x). \quad (3.10)$$

We similarly act with a coordinate transformation on the matter fields; e.g., for a scalar

$$\phi'(x') = \phi(x). \quad (3.11)$$

After changing the integration variable to x' the potential term is modified while the general covariant terms are of course invariant. Given the form (3.10), the modification of the potential can be absorbed in a redefinition of the constants f_{α} . So after the coordinate transformation our action takes the form

$$\begin{aligned} \mathcal{L} = & N \{ \Lambda^2 \sqrt{-g} R(g) - \Lambda^4 V(\tilde{h}') \\ & + \text{higher derivatives} \} + \mathcal{L}_{\text{matter}}(\phi', g) \\ & + \mathcal{O}(N^0), \end{aligned} \quad (3.12)$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}'_{\mu\nu} \quad (3.13)$$

and

$$V(\tilde{h}') = \text{const} + \Lambda^4 \sum_{\alpha=0}^3 (f_{\alpha} n_{(\alpha)}^{\mu} n_{(\alpha)}^{\nu} \tilde{h}'_{\mu\nu})^2 + \mathcal{O}(\tilde{h}'^3). \quad (3.14)$$

We henceforth relabel fields: $\tilde{h}' \rightarrow h$, $\phi' \rightarrow \phi$.

We think of the quadratic terms in the potential as gauge fixing terms, corresponding to the gauge

$$g_{\mu\mu} = \eta_{\mu\mu}, \quad \text{no sum.} \quad (3.15)$$

This defines an acceptable noncovariant gauge. The graviton propagator in this gauge is extremely complicated and un-

wieldy, and so we will not display it here. Fortunately, for leading order calculations we only need to know that it has the structure of the standard covariant propagator

$$\frac{-i}{p^2} (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}) \quad (3.16)$$

plus terms with at least one factor of p (vector) in the numerator. Sandwiched between conserved energy-momentum tensors, the latter p terms vanish, and so we recover the standard Lorentz invariant result. Of course this is no surprise, since Eq. (3.15) represents a valid gauge choice.

The low energy physics is therefore quite similar to what we found in the photon example. We will have two graviton states which propagate at the speed of light, up to a small anisotropic correction. Further, there are four additional Goldstone bosons that acquire kinetic terms at

solutions is hardly surprising given that we are solving (at leading order) second order differential equations.³ Note that the approximately Lorentz invariant solution with constant fields is guaranteed to be an exact solution as it just corresponds to extremizing the potential, while the other solutions with spacetime varying fields need not be exact. In a more conventional field theory context one would expect a solution with time dependent fields to eventually settle down to a static solution by radiating energy. One might expect the same here, with the time dependent fields of the de Sitter type solutions radiating away, leading to the solution with constant fields. A realistic proposal in this framework must involve showing how to make the transition from an expanding radiation or matter dominated universe to the approximately Lorentz invariant solution discussed above. We hope to return to this in future work.

IV. CONCLUDING REMARKS

Building on the work of Bjorken, we have shown that massless photons and gravitons can be produced as Goldstone bosons associated with the spontaneous breaking of Lorentz invariance, and with low energy physics appearing Lorentz invariant to high accuracy. The most dramatic effect of the Lorentz breaking is the existence of additional weakly coupled Goldstone bosons obeying highly non-Lorentz invariant dispersion relations. These fields would be difficult to detect as they couple weakly to conserved currents. A rather general framework for studying Lorentz violating extensions of the standard model has been developed (see, e.g., [12]), and it might be useful to study some of our results in that language.

We find it interesting that the observed low energy physics of gravity can be produced in the context of an effective field theory that differs markedly from general relativity, and which does not suffer from the usual cosmological constant problem. While it remains to be seen whether a theory of this type could be incorporated into a truly fundamental framework or be developed into a realistic cosmology, it seems to be an idea worth pursuing.

ACKNOWLEDGMENTS

This work was supported by NSF grants PHY-0099590 and PHY-9819686.

APPENDIX

In this appendix we indicate one way the gravitational effective action (3.1) may emerge from some underlying conventional dynamics. There may be others. Here we employ the same mechanism as in the photon case, Eq. (2.1). Thus we consider N fermions coupled to gauge fields that acquire masses. We then imagine integrating out the fields from some initial Λ_0 down to a scale Λ obtaining the effective action

$$\bar{\psi}_i(i\partial - M)\psi_i + 4\pi^2 N \sum_k C_k(\Lambda_0, \Lambda) \mathcal{O}_k(\Lambda) \quad (\text{A1})$$

involving an infinite set of fermion interactions. Restricting consideration first to the subset consisting only of powers of

$$\mathcal{O}_{\mu\nu} = \frac{1}{N} \bar{\psi}_i \frac{i}{2} (\gamma_\mu \vec{\partial}_\nu - \gamma_\nu \vec{\partial}_\mu) \psi_i, \quad (\text{A2})$$

we introduce the symmetric auxiliary field $h^{\mu\nu}$ to render the effective action quadratic in the fermions, so that we may write it in the form [cf. Eq. (2.4)]

$$\begin{aligned} \mathcal{L}_{\psi,h} = & (\eta^{\mu\nu} + h^{\mu\nu}) \bar{\psi}_i \frac{i}{2} (\gamma_\mu \vec{\partial}_\nu - \gamma_\nu \vec{\partial}_\mu) \psi_i \\ & - M \bar{\psi}_i \psi_i - N \frac{\Lambda^4}{4\pi^2} V(h). \end{aligned} \quad (\text{A3})$$

All indices are raised and lowered by the flat metric $\eta_{\mu\nu}$.

Integrating out the fermions, the effective action from the resulting determinant can be expressed, as usual, as the sum over all fermion one loop diagrams with external h legs. Note, in particular, that the diagram with one external h leg is in general nonvanishing. This reflects the fact that Eq. (A2) has a nonvanishing expectation (proportional to $\eta_{\mu\nu}$) even in ordinary perturbation theory on a Lorentz invariant vacuum, i.e., interactions built from Eq. (A2) shift the classical background. Correspondingly, all terms, including a linear term, are included in the general potential $V(h)$ in Eq. (3.1) (cf. discussion in the text).

Explicit evaluation of the fermion-loop graphs with one and two external h legs gives, after a lengthy computation, the contribution to the effective action to $\mathcal{O}(h^2)$:

$$\begin{aligned} \mathcal{L}^{(2)} = & NI_4 \left[-h^\mu_\mu + \frac{1}{2} h^{\mu\nu} h_{\mu\nu} + \frac{1}{2} (h^\mu_\mu)^2 \right] + \frac{N}{6} I_2 [(\partial_\lambda h^{\mu\nu})^2 - (\partial$$

Equation (A4) displays explicitly the leading terms, i.e., the local, cutoff dependent (“divergent”) part of the result of the loop integrations. The ellipses denote the subleading pieces from the finite, nonlocal parts, which can be expanded in powers of $\Box/M^2$, and contribute to higher derivative interactions relevant only for short distance behavior near the cutoff. Dimensional regularization, under the usual correspondence $\ln \Lambda \leftrightarrow (1/\epsilon + \text{const})$, gives

$$I_n = \frac{1}{(4\pi)^2} M^n \ln \left(\frac{\Lambda^2}{M^2} \right), \quad n=0,2,4. \quad (\text{A5})$$

In Pauli-Villars regularization, which appears more physical in the present context, one has

$$I_n = \frac{1}{(4\pi)^2} \sum_{k=1}^3 c_k M_k^n \ln \left(\frac{M_k^2}{M^2} \right), \quad n=0,2,4. \quad (\text{A6})$$

Three regulator masses M_k , of order of the cutoff Λ , are required here with coefficients c_k satisfying $\sum_{k=1}^3 c_k M_k^n + M^n = 0$, for $n=0, 2, 4$.

Equation (A4) is now seen to be the flat space expansion to second order of the gravitational action

$$\mathcal{L} = \sqrt{-g} N \left[I_4 + \frac{1}{6} I_2 R - \frac{1}{20} I_0 \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) \right] \quad (\text{A7})$$

with the metric expressed in terms of the vierbein⁴

$$g^{\mu\nu} = e_a^\mu e^{a\nu}, \quad e_a^\mu = \delta_a^\mu + h_a^\mu, \quad (\text{A8})$$

and

$$h^{\mu\nu} = \delta_a^\mu h^{a\nu}. \quad (\text{A9})$$

Taking $h^{\mu\nu}$ to be symmetric, as done in the above calculation, amounts to the (standard) local Lorentz gauge fixing to a symmetric vierbein. [It is known that the antisymmetric part in fact decouples in Eq. (A7).] Thus our effective gravitational action (3.1) is reproduced to this order.

To see how this comes about, note that the result (A4) is precisely what one obtains after integrating out the fermion fields in the Lagrangian for N fermions in curved space:

$$\mathcal{L} = e^{(1-2w)} \left[e_a^\mu \bar{\psi}_i \frac{i}{2} (\gamma^a \vec{\nabla}_\mu - \vec{\nabla}_\mu \gamma^a) \psi_i - M \bar{\psi}_i \psi_i \right], \quad (\text{A10})$$

to second order in the expansion about flat space (A8). In Eq. (A10), $e = \det e_{a\mu}$, and

⁴Curved and flat indices are denoted by Greek and Latin letters, respectively.

$$\vec{\nabla}_\mu \equiv \partial_\mu + \frac{i}{2} \omega_{\mu ab} S^{ab} \quad (\text{A11})$$

with Lorentz generators $S^{ab} = (i/4)[\gamma^a, \gamma^b]$. The spinor ψ may, in general, be taken to transform under general coordinate transformations as a density of weight w . The result of the explicit computation is in fact found to be independent of w (see, for example, [13]). Indeed, note that, in terms of the functional integration, the factor $e^{(1-2w)}$ can be absorbed by the change of variables: $\psi \rightarrow e^{(1-2w)/2} \psi'$, $\bar{\psi} \rightarrow e^{(1-2w)/2} \bar{\psi}'$. The spin connection $\omega_{\mu ab}$ is given in terms of the vierbein by

$$\omega_{\mu ab} = \frac{1}{2} e_\mu^a (T_{amn} - T_{mna} - T_{nam}),$$

$$T_{mn}^a = (e_m^\rho e_n^\sigma - e_n^\rho e_m^\sigma) \partial_\$$

ing independent auxiliary field $\omega_{\kappa\lambda}^\mu$, Eq. (A3) is extended to

$$\begin{aligned} \mathcal{L}_{\psi,h} = & (\eta^{\mu\nu} + h^{\mu\nu}) \bar{\psi}_i \frac{i}{2} \\ & \times \left(\gamma_\mu \tilde{\partial}_\nu - \gamma_\mu \tilde{\partial}_\nu + \frac{1}{8} \omega_{\nu\kappa\lambda} \{ \gamma_\mu, [\gamma^\kappa, \gamma^\lambda] \} \right) \psi_i \\ & - M \bar{\psi}_i \psi_i - N \frac{\Lambda^4}{4\pi^2} V(h, \omega). \end{aligned} \quad (\text{A14})$$

The fermionic part of Eq. (A14) is equivalent to Eq. (A10) in the first order (Palatini) formulation.

Now, in the first order formalism, the nonpropagating connection in Eq. (A10) serves as a constraint field enforcing vanishing of torsion $e_m^\mu \mathcal{O}^{mab}$ generated by the fermions.

Equation (A14) differs from Eq. (A10) by the presence of a potential in ω . Hence, variation of the connection will not imply vanishing torsion, and ω will not be expressible en-

tirely in terms of the vierbein as in Eq. (A12).⁶ For illustration purposes, we may adopt a model where the quadratic terms in the potential for ω are suppressed. Then, upon integrating out the fermions, Eq. (A7) and the effective gravitational action (3.1) are reproduced to within small deviations. This is easily seen to be stable under radiative corrections from graviton loops.

It is perhaps worth pointing out again that, as we saw, just the fermionic self-interactions of products of Eq. (A2) in flat spacetime already suffice to fully reproduce to second order the Einstein-Hilbert (plus R^2 terms) parts in our effective gravitational action (3.1), i.e., reproduce the full content of linearized general relativity.

⁶This is analogous to considering Eq. (A10) in the first order formalism not just by itself, but with the addition of the Einstein-Hilbert action. The latter provides a potential for the connection ω , thus leading to the usual result of torsion generated by fermions. This is, of course, the usual situation. In the above we were led to consider Eq. (A10) by itself in the context of generating the Einstein-Hilbert action from Eq. (A1).

-
- [1] J. D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963).
 - [2] G. S. Guralnik, Phys. Rev. **136**, B1404 (1964).
 - [3] P. R. Phillips, Phys. Rev. **146**, 966 (1966).
 - [4] H. C. Ohanian, Phys. Rev. **184**, 1305 (1969).
 - [5] D. Atkatz, Phys. Rev. D **17**, 1972 (1978).
 - [6] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
 - [7] S. Weinberg and E. Witten, Phys. Lett. **96B**, 59 (1980).
 - [8] J. Bjorken, hep-th/0111196.

- [9] T. Banks and A. Zaks, Nucl. Phys. **B184**, 303 (1981).
- [10] S. Kachru, M. Schulz, and E. Silverstein, Phys. Rev. D **62**, 045021 (2000).
- [11] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, Phys. Lett. B **480**, 193 (2000).
- [12] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
- [13] D. M. Capper and M. J. Duff, Nucl. Phys. **B82**, 147 (1974).